Equations that she discovered

- 2019, July

$$3\sqrt{375} = \sqrt{3375},\ 9\sqrt{1125} = \sqrt{91125}.$$

- 2019, July

$$\sqrt{3} = 1 + \sqrt{\cfrac{1}{1 + \sqrt{\cfrac{1}{1 + \cfrac{1}{3}}}}},\ \sqrt{45} = 6 + \sqrt{\cfrac{1}{1 + \sqrt{\cfrac{1}{1 + \cfrac{1}{80}}}}}.$$

REMARK 0.1. She found the above equation the day after I (Yuki) taught her continued fraction expansion.

REMARK 0.2. (By Hiroaki Ito, Tsukuba University) It is possible to construct infinitely many similar equations using Pell's equation (the argument is nontrivial). For example, we have

$$\sqrt{1275} = 35 + \sqrt{\cfrac{1}{1 + \sqrt{\cfrac{1}{1 + \cfrac{1}{2499}}}}},\ \sqrt{41905} = 204 + \sqrt{\cfrac{1}{1 + \sqrt{\cfrac{1}{1 + \cfrac{1}{83520}}}}}.$$

- 2019, July

$$\sqrt{2} = 1 + \cfrac{1 + \cfrac{1 + \cfrac{1+\cdots}{3+\cdots}}{3 + \cfrac{1+\cdots}{3+\cdots}}}{3 + \cfrac{1 + \cfrac{1+\cdots}{3+\cdots}}{3 + \cfrac{1+\cdots}{3+\cdots}}}$$

- 2019, July

$$\frac{20!7!}{6!21!} = \frac{207}{621}, \quad \frac{175!56!}{55!176!} = \frac{17556}{55176}, \quad \frac{1500!475!}{474!1501!} = \frac{1500475}{4741501}$$

Remark 0.3. (By Masataka Kanki, Kansai University)

$$\frac{29600!9361!}{9360!29601!} = \frac{296009361}{936029601}, \quad \frac{253075!80030!}{80029!253076!} = \frac{25307580030}{80029253076}.$$

- 2019, July

$$1 + 1 + 2 + 3 + 5 + 8 + 13 + \cdots = -1.$$

Remark 0.4. This was already known. This can be "shown" by considering the mother function of the Fibonacci sequence and plugging in $x = 1$.

- 2019, July (she says that she is confident that there exists a formula to yield prime numbers!!!)

「nのフィールド上にあると仮定した軸を持った螺旋にそれと正反対の回転と角度を持った螺旋を任意の(適当な？)CBRにそって回転させていって、それがハマれば当たり」

＊上が彼女の「素数を求める方法」
＊CBRはキャプティブビーズリングの略でボディーピアスのこと。

- 2019, August

$$57 \cdot 56 \cdot 55 = 22 \cdot 21 \cdot 20 \cdot 19.$$

Remark 0.5. (By Masataka Kanki, Kansai University) The non-trivial solution of

$$x(x+1)(x+2) = y(y+1)(y+2)(y+3)$$

is only $(x, y) = (55, 19)$ for $y < 10000$.

- 2019, August

$$2 = \sqrt[3]{3 + \frac{11}{3}\sqrt{\frac{2}{3}}} + \sqrt[3]{3 - \frac{11}{3}\sqrt{\frac{2}{3}}}$$

Remark 0.6. (By Masataka Kanki, Kansai University) The above equation shows up when you solve

$$x^3 - x - 6 = 0$$

using Cardano's formula.

- 2019, August

$$\pi^2 = \cfrac{12}{1 + \cfrac{1^4}{3 + \cfrac{2^4}{5 + \cfrac{3^4}{7 + \cfrac{4^4}{9 + \cdots}}}}}$$

Remark 0.7. This expression was already known (unfortunately!).

- 2019, August

$$11! = (16 \cdot 15) \cdot (56 \cdot 55 \cdot 54).$$

- 2019, August

$$333 - 3!3!3! = 141 - 1!4!1!.$$

- 2019, August

$$90 \cdot 88 = 22 \cdot 20 \cdot 18.$$

- 2019, August

$$66 \cdot 65 \cdot 64 \cdot 63 = 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8.$$

- 2019, August

$$(5!11!)^2 + (4!12!)^2 = (2!13!)^2.$$

- 2019, August

$$\tan\frac{\pi}{9} + 4\sin\frac{\pi}{9} = \sqrt{3}.$$

- 2019, August

$$\frac{\log_4 4 \cdot \log_4 4}{\log_2 16 \cdot \log_2 2} = \frac{4^4 \cdot 4^4}{2^{16} \cdot 2^2}.$$

- 2019, August

$$3!5!7! = 10!.$$

From below we use the notation $n!\cdots!$ in such a way that, for example,

$$100!! = 100 \cdot 98 \cdots 2,\ 100!!! = 100 \cdot 97 \cdots 1.$$

- 2019, August

$$2!!3!!4!! = 6!!.$$

- 2019, August

$$1!2!3!4!5! = 2!4!6!.$$

- 2019, August

$$6!!!!7!!8!!9!!!! = 10!.$$

- 2019, August

$$5!!6!!!!7!!!!!!8!!!9!!!!! = 10!.$$

- 2019, August

$$5!!!6!!7!!!!!!8!!!!!9!!!! = 10!.$$

- 2019, August

The area of the triangle whose length of the three sides are $61, 80, 109$ (resp. $\sqrt{61}, \sqrt{80}, \sqrt{109}$) is 2400 (resp. 34).

Remark 0.8. (By Masataka Kanki, Kansai University) The above is already known. It is possible to construct infinitely many triangles with the same property ($\{360, 676, 964\}, \{901, 1621, 1872\}, \cdots$).

- 2019, August (to be more precise, 2019/08/23)

$$20190823,\ 190823,\ 90823,\ 823,\ 23,\ 3$$

are all prime numbers. The dates 2030/03/17, 2019/05/23 have the same property.

- 2019, August

$$\sum_{k=2n^2+n}^{2n^2+2n} k^2 = \sum_{k=2n^2+2n+1}^{2n^2+3n} k^2.$$

Remark 0.9. She did not know $\sum$ notation until 2019, August!!!